Motion and Structure from Event-based Normal Flow

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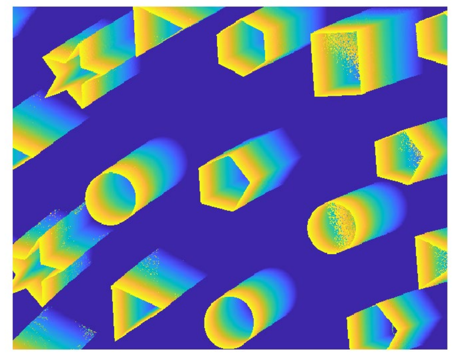
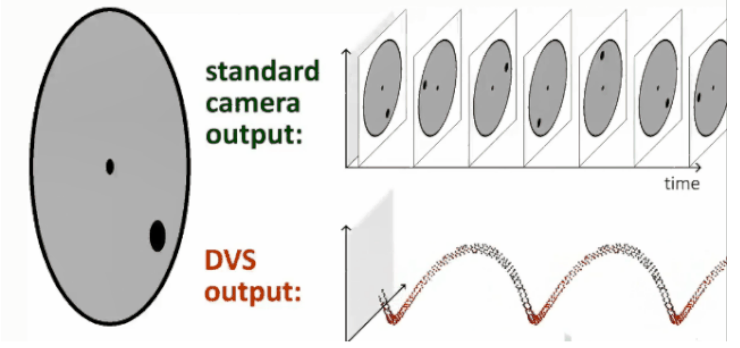
NAIL Neuromorphic Automation & Intelligence Laboratory

Project Page: https://nail-hnu.github.io/EvLinearSolver

Motivation

Event Camera





- ☐High temporal resolution
- ☐High dynamic range
- ☐Low latency
- ☐Low power consuming

Existing Methods for Event-based Motion-Model Fitting

Representative work:
- ➢ CMax [Gallego, CVPR 2018]
- ➢ Pro-STR [Huang, CVPR 2023]

Limitations:
- ☐Require a proper initialization
- ☐Rely on the constant-velocity assumption

Contribution

➢ **A normal flow constraint** with a geometric connection to event data.
It overcomes the partial observability issue.

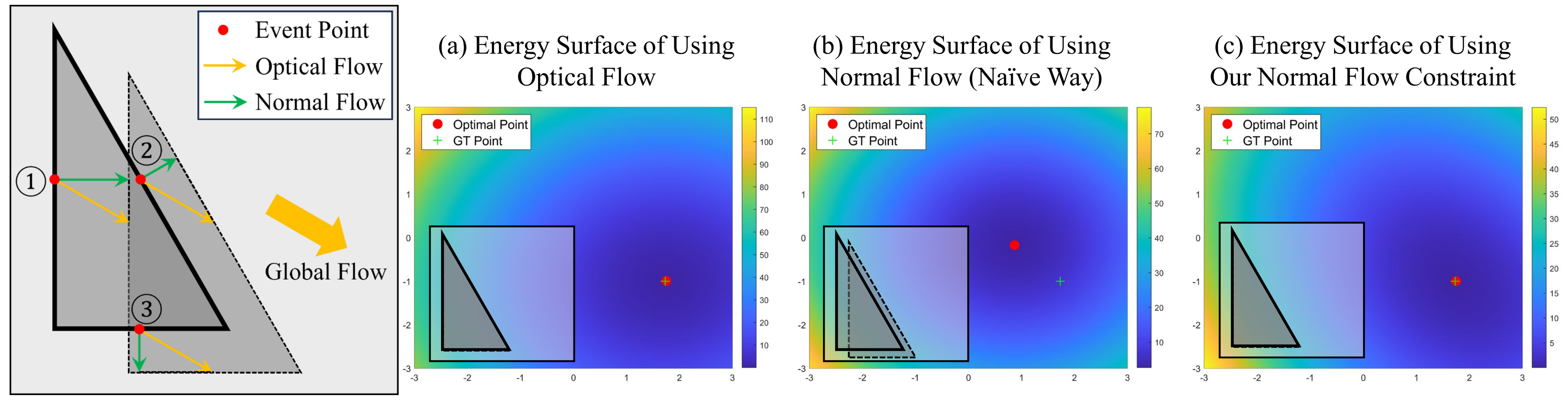
➢ **Two solvers** for a family of motion-and-structure estimation problems.
1. Linear solver: integrated with RANSAC, **an initialization to existing nonlinear methods.**
2. Nonlinear solver: under a continuous-time formulation, **free of the constant-motion assumption.**

➢ A thorough evaluation on the two proposed solvers
1. Performance comparison against state-of-the-art methods
2. An investigation on the performance of existing nonlinear methods initialized using our linear solver.

Normal Flow Constraint

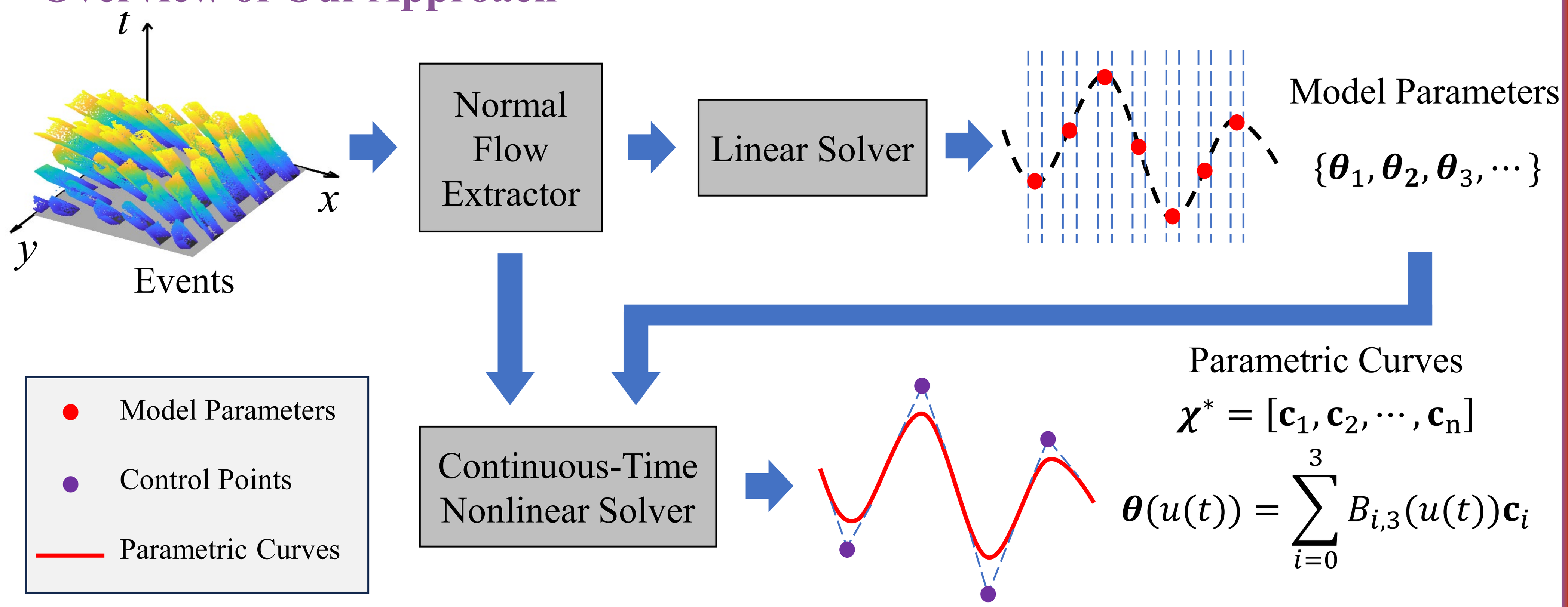
$$\mathbf{n}(\mathbf{x})^\top \mathbf{u}(\mathbf{x};\boldsymbol{\theta}) - \|\mathbf{n}(\mathbf{x})\|^2 \doteq \epsilon_{\mathrm{nf}} \in \mathbb{R}$$

Observation: Normal Flow



Methodology

Overview of Our Approach



Linear Solver

RANSAC

$$\begin{pmatrix} \mathbf{n}(\mathbf{x}_1)^\top \mathbf{O}(\mathbf{x}_1) \\ \vdots \\ \mathbf{n}(\mathbf{x}_K)^\top \mathbf{O}(\mathbf{x}_K) \end{pmatrix} \boldsymbol{\theta} = \begin{pmatrix} \|\mathbf{n}(\mathbf{x}_1)\|^2 \\ \vdots \\ \|\mathbf{n}(\mathbf{x}_K)\|^2 \end{pmatrix}$$

Continuous-Time Nonlinear Solver

Nonlinear Least Square

$$\mathcal{X}^* = \arg\min_{\mathcal{X}} \sum_{e_k \in \mathcal{E}} \left\| \mathbf{n}(\mathbf{x}_k)^\top \mathbf{O}(\mathbf{x}_k) \boldsymbol{\theta}(u(t_k)) - \|\mathbf{n}(\mathbf{x}_k)\|^2 \right\|^2$$

Applications

Angular Velocity Estimation

$$\mathbf{u}(\mathbf{x}) = \mathbf{B}(\mathbf{x})\boldsymbol{\omega}$$

(Motion Flow Equation, Trucco, 1998)

$$\mathbf{n}(\mathbf{x})^\top \mathbf{B}(\mathbf{x})\boldsymbol{\omega} = \|\mathbf{n}(\mathbf{x})\|^2$$

6 DoF Motion Tracking

$$\mathbf{u}(\mathbf{x}) = \frac{\mathbf{A}(\mathbf{x})\boldsymbol{v}}{Z(\mathbf{x})} + \mathbf{B}(\mathbf{x})\boldsymbol{\omega}$$

(Motion Flow Equation, Trucco, 1998)

$$\mathbf{n}(\mathbf{x})^\top \mathbf{D}(\mathbf{x}) \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix} = \|\mathbf{n}(\mathbf{x})\|^2$$

Differential Homography

$$\hat{\mathbf{u}}(\mathbf{x}) = (\mathbf{1} - \hat{\mathbf{x}}\mathbf{e}_3^\top)\mathbf{H}_d\hat{\mathbf{x}}$$

(Differential Homography, Ma, 2004)

$$\mathbf{n}(\mathbf{x})^\top \mathbf{C}(\mathbf{x})\mathbf{h}_d = \|\mathbf{n}(\mathbf{x})\|^2$$

Experiments

Qualitive Results on Optical Flow and Depth

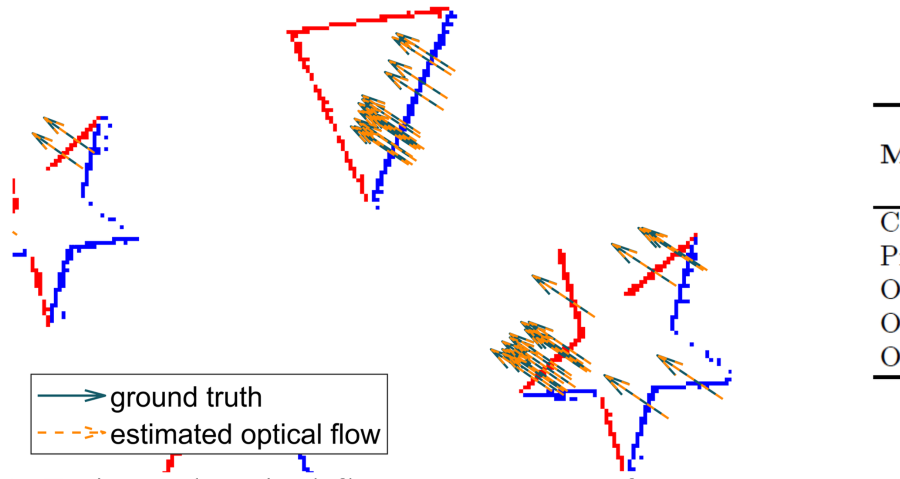


Fig. 1: Estimated optical flow over a map of event accumulation.

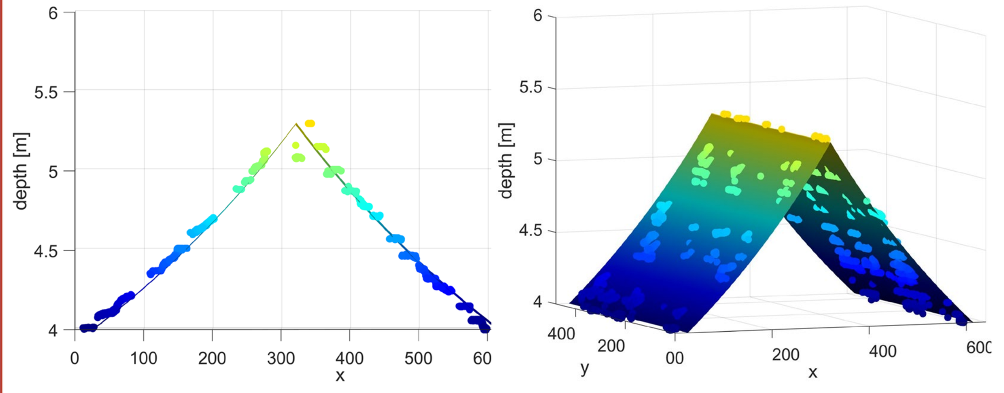


Fig. 2: Reconstruction result displayed over groundtruth structures (a v-shape roof) and viewed from different perspectives.

Quantitative Results on Motion Estimation Tasks

Table 1: Evaluation of our linear solver on the task of angular velocity estimation

Method	*shapes_rotation*		*ground_rotation* (our data)		*patterns_rotation* (our data)	
	e_ω (deg/s)	RMSE$_\omega$ (deg/s)	e_ω (deg/s)	RMSE$_\omega$ (deg/s)	e_ω (deg/s)	RMSE$_\omega$ (deg/s)
CMax	15.60	26.38	9.48	15.58	5.95	9.07
Pro-STR	90.61	145.81	31.07	44.74	10.47	11.88
Ours	<u>15.36</u>	<u>23.98</u>	<u>3.30</u>	<u>8.01</u>	4.70	6.08
Ours+CMax	**10.12**	**14.29**	**2.85**	**3.74**	**0.35**	**0.73**
Ours+Pro-STR	19.23	40.44	9.13	17.07	<u>4.31</u>	<u>5.23</u>

Table 2: Evaluation of our linear solver on the task of differential homography

Method	*board_slow*	*patterns_rotation*	*patterns_6dof*
	F-Norm	F-Norm	F-Norm
CMax	1.16	1.33	1.27
Pro-STR	2.07	1.22	1.37
Ours	0.24	0.20	0.43
Ours+CMax	<u>0.23</u>	**0.15**	**0.41**
Ours+Pro-STR	**0.19**	<u>0.17</u>	<u>0.42</u>

Table 3: Evaluation of our linear solver on the task of 6-DoF motion tracking

Method	*corner_slow*				*patterns_6dof* (our data)			
	e_ω (deg/s)	RMSE$_\omega$ (deg/s)	e_v (m/s)	RMSE$_v$ (m/s)	e_ω (deg/s)	RMSE$_\omega$ (deg/s)	e_v (m/s)	RMSE$_v$ (m/s)
CMax	12.39	14.39	0.34	0.41	12.94	15.68	1.02	1.23
Pro-STR	5.19	7.84	0.13	0.22	11.21	13.49	1.33	1.49
Ours	<u>1.31</u>	<u>1.79</u>	**0.01**	**0.02**	<u>0.56</u>	<u>0.89</u>	**0.12**	<u>0.16</u>
Ours+CMax	2.19	2.82	0.11	0.17	**0.55**	**0.87**	0.18	0.30
Ours+Pro-STR	**1.06**	**1.34**	<u>0.04</u>	<u>0.05</u>	1.07	1.56	**0.12**	**0.15**

Qualitive Results on Continuous-Time Motion Estimation

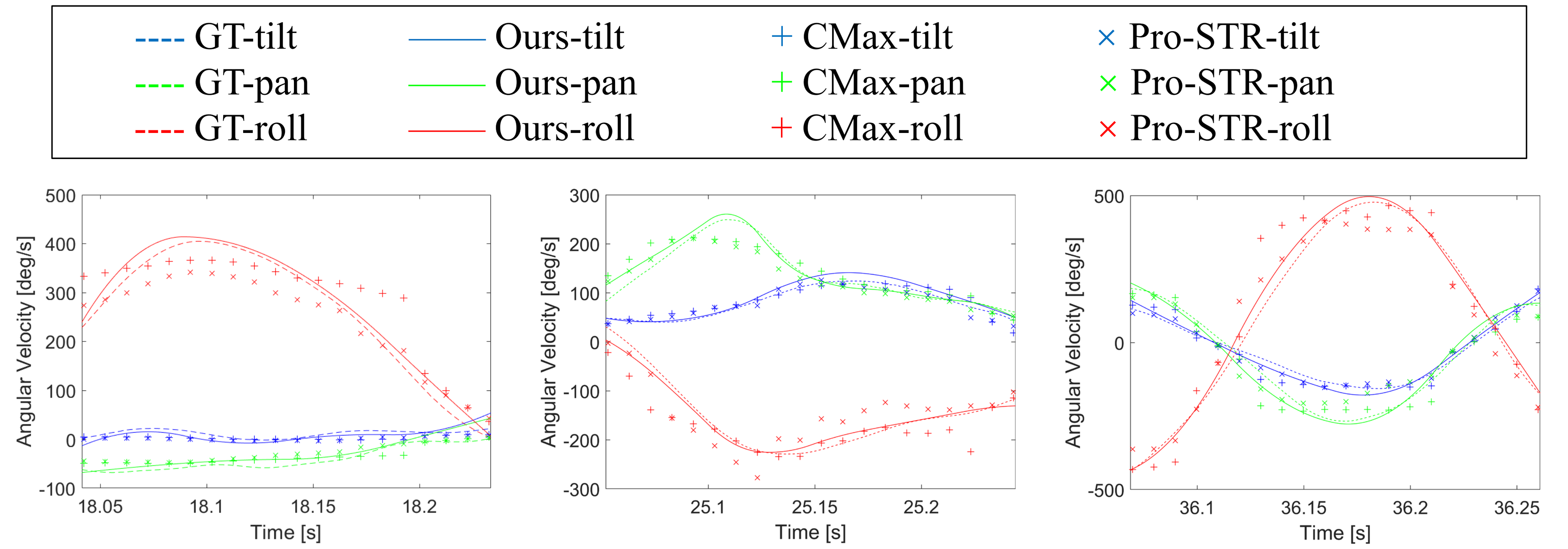


Fig. 3: Comparison against two state-of-the-art nonlinear solutions using data segments that involve sudden variations of angular velocity

Seeking a Ph.D Position for Fall 2025!